Zeroing in on More Photons and Gluons

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Abstract

We discuss radiation zeros that are found in gauge tree amplitudes for processes involving multi-photon emission. Previous results are clarified by examples and by further elaboration. The conditions under which such amplitude zeros occur are identical in form to those for the single-photon zeros, and all radiated photons must travel parallel to each other. Any other neutral particle likewise must be massless (e.g. gluon) and travel in that common direction. The relevance to questions like gluon jet identification and computational checks is considered. We use examples to show how certain multi-photon amplitudes evade the zeros, and to demonstrate the connection to a more general result, the decoupling of an external electromagnetic plane wave in the "null zone". Brief comments are made about zeros associated with other gauge-boson emission.

I. Introduction

It is now more than a dozen years since radiation amplitude zeros were first discovered [1] in the process $u\overline{d} \rightarrow W^+\gamma$ [2]. Subsequently, it was shown [3, 4, 5] that these can arise more generally, originating as the destructive interference of radiation patterns in gauge-theory tree amplitudes for massless gauge-boson emission. This is therefore a property of gauge theories; anomalous electromagnetic moments, for example, would spoil the perfect cancellations and such anomalies are forbidden in gauge couplings. For a specific analysis of the effect of W anomalous moments in the $u\overline{d} \rightarrow W^+\gamma$ reaction, see [6]. Of course, anomalous moments come up in higher-order corrections, and indeed radiation zeros do not appear beyond the tree approximation in any theory.

Can we observe these zeros experimentally? Since we must be able to use the tree approximation, the couplings have to be small for the process considered. The equations that determine where the zeros are - essentially these are just the demand that the ratio of coupling to light-cone energy be the same for all particles - specifically require that all couplings have the same sign (electric charges, for photon emission). These two constraints, weak couplings and same-sign charges, have very much limited the number of reactions in which a RdipS would be found. In high-energy quark reactions where gluon emission can be described perturbatively, the color charges, unfortunately, are ultimately averaged or summed over. The benchmark e^+e^- reactions violate the like-sign condition. Even if certain hadronic reactions involved electric charges of only one sign, and in some limit could be approximated by tree amplitudes, hadrons with spin are composite particles with anomalous

moments ($g \neq 2$). And small cross sections are perforce hard to measure!

Despite the difficulty in "measuring" zeros in experiments, the sensitivity of the basic quark amplitude $u\overline{d} \to W^+\gamma$ to the W-boson magnetic moment has attracted much interest in the hope that this important parameter could be measured in proton-antiproton colliders [7]. The radiation zero is present only for a magnetic moment value corresponding to $g = 2$ as predicted by gauge theory. The extent to which there is a pronounced dip will make it possible to put limits on the W magnetic moment (see Sec. V); experimenters can also consider the crossed channel reaction, radiative W-decay, whose zero shows up in the energy distributions [8]. A basic $e^-e^- \to e^-e^-\gamma$ radiation zero is irrelevant to present accelerators, but there is the possibility that HERA experiments may probe a radiation amplitude zero in electron-quark bremsstrahlung and allow a direct measurement of the quark charge [9, 10, 11].

The very welcome progress in accelerator experimentation brings with it a challenge. We can anticipate more detailed information, not only in the way of more single-photon events, but also in the variety of final states measured. Are there electromgnetic radiation zeros in reactions with more final photons, such as two-photon exclusive reactions? What about the associated production in the QCD perturbative regime of another massless gauge boson of great interest, the gluon, such as in $u\overline{d} \to W^+\gamma g$?

The answer to both is that, yes, in general the radiation zeros survive the addition of more neutral, massless particles. If a given reaction has an electromagnetic radiation zero in its tree amplitude, then the tree reaction with additional photons and gluons produced

in the final state will too, occurring when these additional particles travel parallel to the original photon, sharing its original energy. This answer is given to us already in Ref. [5].

In the present paper, we revisit this question, in view of the experimental change of scenery and the fact that the higher-order zeros are not so well known. The purpose is to call attention to our previous results on multi-photon/gluon electromagnetic radiation zeros, and to try to combine them into a self-contained and clear picture through detailed examples. The examples are laid out in Sec. II. In the next two sections, we use the examples to illustrate the survival theorem [5] for neutral, massless particles, and the decoupling theorem [12, 13] for an external electromagnetic plane wave field. Finally, we consider the relevance of the multi-gauge-boson emission zeros as they pertain to the new generation of electron-proton and proton-antiproton colliding beams.

II. Examples

We wish to present several tree amplitudes for the emission/absorption of multiple gauge bosons. The first intention is to exhibit their

zero structure, and also to show a simple counter-example, in which there are multiple photons but neither a physical nor unphysical radiation zero. Second, we focus on a reaction relevant to experiment.

A. Scalar particles and photons

Consider the radiative process where a scalar particle decays in lowest order through a single-vertex scalar interaction into $n-1$ other scalar particles plus one photon. Denoting the electric charges by Q_i, the reaction is

$$Q_1 \;\rightarrow\; Q_2 \,+\ldots+\, Q_n \,+\, \gamma(q)$$

The diagrams of the tree amplitude are illustrated in Fig. 1 and we can write it as

$$M = \sum_{i=1}^{n} \left(\frac{Q_i}{p_i \cdot q} - \frac{Q_j}{p_j \cdot q} \right) g \delta_i p_i \cdot \epsilon \tag{2.1}$$

where j is fixed and can take any i value, and $\delta_i = -1(+1)$ for incoming (outgoing) particles.

The fact that we could rearrange (factorize) (2.1) as shown [14, 3, 4, 5] is due to the presence of the zero. It is evident that the amplitude vanishes in the null zone defined by the $n-1$ equations [these actually reduce to $n-2$ independent ones by charge and momentum conservation]

$$\frac{Q_i}{p_i \cdot q} \;=\; \frac{Q_j}{p_j \cdot q}, \quad \text{all } i,\ j \tag{2.2}$$

That is, the kinematical conditions for the null radiation zone are that all particles must have the same charge to light-front-energy ratio. The ratios are recognized as the factors arising from the attachment of a photon to the various external lines, with the remarkable feature that these conditions also suffice to cancel out internal-line attachments in the tree amplitudes. (One sees immediately why closed-loop, higher-order amplitudes will not be nullified: Integrated internal loop momenta are certainly not fixed.)

We digress for the moment. Recall that the radiation amplitude zero is not spoiled by photons attached to internal tree lines. For example, if we look at a tree source graph with one internal line, the photon attachments can be rearranged into a sum over two vertex terms. The vertex terms can themselves be rearranged as in (2.1). In particular, consider the process

$$Q_1 \to Q_2 + (Q_{int} \to Q_3 + \ldots + Q_n) + \gamma(k)$$

where $Q_{int} = Q_1 - Q_2$ represents a virtual particle of mass m. The total amplitude for this process is obtained by attaching the photon in all possible ways to the external lines and also the internal line (see Fig. 2). Using the radiation decomposition identity [3, 5] on the term with photon emission from the internal line ($p' = p - q$)

$$\frac{1}{p'^2 - m^2} Q(p' + p) \cdot \epsilon \frac{1}{p^2 - m^2} = \frac{1}{p'^2 - m^2} \frac{Q}{p' \cdot q} p' \cdot \epsilon - p \cdot \epsilon \frac{Q}{p \cdot q} \frac{1}{p^2 - m^2} \tag{2.3}$$

this amplitude can be written as two clusters corresponding to corrections to the two source vertices

$$\begin{aligned} M = \; & -i\epsilon^* \cdot \Big\{ \Big[\tfrac{Q_1 p_1}{p_1 \cdot q} - \tfrac{Q_2 p_2}{p_2 \cdot q} - \tfrac{Q_{int}(p_1 - p_2)}{(p_1 - p_2) \cdot q} \Big] \tfrac{1}{(p_1 - p_2 - q)^2 - m^2} \\ & - \Big[\textstyle\sum_{i=3}^{n} \tfrac{Q_i p_i}{p_i \cdot q} - \tfrac{Q_{int}(p_1 - p_2)}{(p_1 - p_2) \cdot q} \Big] \tfrac{1}{(p_1 - p_2)^2 - m^2} \Big\} \end{aligned} \tag{2.4}$$

Since the quantity in each square bracket vanishes under the zero conditions, we see that the zeros persist at the same location in phase space, independent of the mass of the internal particle. In its clustered form this example will help the reader follow the more general discussion given in Sec. III.

Now add another photon. The process we consider is

$$Q_1 \;\to\; Q_2 \;+ \ldots +\; Q_n \;+\; \gamma(q_1) \;+\; \gamma(q_2).$$

Although we still restrict ourselves to spinless charges with scalar self-interactions, the subsequent discussion will make it clear how spin and gauge interactions can be incorporated. We again consider only a single n-scalar vertex.

It is not hard to write down the lowest-order tree amplitude such that the zero is manifest. We can rearrange the sum of diagrams, using experience gained from our previous factorization study to rewrite some of the terms,

$$\begin{aligned} M \;=\; & g \sum_{i=1}^{n} \tfrac{\delta_i Q_i}{p_i \cdot q_1} p_i \cdot \epsilon_1^* \sum_{k=1}^{n} \left(\tfrac{Q_k}{p_k \cdot q_2} - \tfrac{Q_j}{p_j \cdot q_2} \right) (\delta_k p_k + \delta_{ik} q_1) \cdot \epsilon_2^* \\ & + g \sum_{i=1}^{n} \left(\tfrac{Q_i}{p_i \cdot (q_1+q_2)} - \tfrac{Q_j}{p_j \cdot (q_1+q_2)} \right) \delta_i Q_i (-\epsilon_1^* \cdot \epsilon_2^*) \\ & + q_1 \cdot q_2 g \sum_{i=1}^{n} Q_i^2 \Big[\tfrac{1}{\delta_i p_i \cdot (q_1+q_2) + q_1 \cdot q_2} \tfrac{1}{p_i \cdot q_2} \tfrac{1}{(\delta_i p_i + q_1) \cdot q_2} p_i \cdot \epsilon_1^* \, (\delta_i p_i + q_1) \cdot \epsilon_2^* \\ & \quad - \tfrac{1}{p_i \cdot q_1} \tfrac{1}{p_i \cdot q_2} \tfrac{1}{(\delta_i p_i + q_1) \cdot q_2} \delta_i p_i \cdot \epsilon_1^* \, (\delta_i p_i + q_1) \cdot \epsilon_2^* \\ & \quad + \tfrac{1}{\delta_i p_i \cdot (q_1+q_2)} \tfrac{1}{\delta_i p_i \cdot (q_1+q_2) + q_1 \cdot q_2} \epsilon_1^* \cdot \epsilon_2^* \Big] \\ & + g \sum_{i=1}^{n} \tfrac{Q_i^2}{\delta_i \, p_i \cdot (q_1+q_2) + q_1 \cdot q_2} \tfrac{1}{p_i \cdot q_2} (q_2 \cdot \epsilon_1^* p_i \cdot \epsilon_2^* - q_1 \cdot \epsilon_2^* \, p_i \cdot \epsilon_1^*) \end{aligned} \tag{2.5}$$

On the face of it, one might assume that we need three different sets of conditions for (2.5) to vanish. The first set is the null zone conditions for q_1

$$\frac{Q_i}{p_i \cdot q_1} = \frac{Q_j}{p_j \cdot q_1}, \quad \text{all } i,\ j \tag{2.6}$$

The second set is the analogous conditions for q_2

$$\frac{Q_i}{p_i \cdot q_2} = \frac{Q_j}{p_j \cdot q_2}, \quad \text{all } i,\ j \tag{2.7}$$

And the third is that the two photons must be parallel (their momenta must be proportional to the same null vector, $q_1,\ q_2 \propto n$).

Actually, one set of null zone conditions is all we need. From the conditions (2.6), for example, taken alone, it follows that the second photon, with its zero charge, must be

massless and parallel to the first photon, and therefore the set (2.7) follows as well. We refer the reader to Sec. III and a related discussion in [5]. **Equation (2.6) is therefore sufficient**.

B. Photons and Gluons

Next we look at an example very much relevant to experiment. This will serve to introduce spin, another massless neutral particle, and a parton reaction to which we return later in the paper. Consider the radiative decay process where quark-antiquak annihilation leads to a W-boson plus a gluon plus a photon,

$$u\overline{d} \;\rightarrow\; W^+ \,+\, g \,+\, \gamma$$

The eight diagrams of its tree amplitude are indicated in Fig. 3. Drawing again on our previous experience[5], we can collapse the results into the following form

$$\begin{aligned}
M \;&= ieG^{ud}_{V-A}\bar{v}(p_2)\Big\{\tfrac{Q_{2,color}}{2p_2\cdot k}\Big[\textstyle\sum_{i=1}^{3}\Big(\tfrac{Q_i}{p_i\cdot q}-\tfrac{Q_j}{p_j\cdot q}\Big)\,\delta_i p_i\cdot\epsilon_q\,(\not{k}\;\not{\epsilon}_k+2p_2\cdot\epsilon_k)\;\not{\epsilon}_3 \\
&+\Big(\tfrac{Q_1}{p_1\cdot q}-\tfrac{Q_3}{p_3\cdot q}\Big)\,(\not{k}\;\not{\epsilon}_k\;\not{\epsilon}_q\,q\cdot\epsilon_3-2\;\not{q}\,\epsilon_q\cdot\epsilon_3\,p_2\cdot\epsilon_k+2\;\not{\epsilon}_q\,q\cdot\epsilon_3\,p_2\cdot\epsilon_k) \\
&+\Big(\tfrac{Q_1}{p_1\cdot q}-\tfrac{Q_2}{p_2\cdot q}\Big)\;\not{q}\;\not{\epsilon}_q\,p_2\cdot\epsilon_k\;\not{\epsilon}_3\Big] \\
&-\tfrac{Q_{1,color}}{2p_1\cdot k}\Big[\textstyle\sum_{i=1}^{3}\Big(\tfrac{Q_i}{p_i\cdot q}-\tfrac{Q_j}{p_j\cdot q}\Big)\,\delta_i p_i\cdot\epsilon_q\;\not{\epsilon}_3\,(\not{\epsilon}_k\;\not{k}+2p_1\cdot\epsilon_k) \\
&+\Big(\tfrac{Q_2}{p_2\cdot q}-\tfrac{Q_3}{p_3\cdot q}\Big)\,(\not{\epsilon}_q\;\not{\epsilon}_k\;\not{k}\,q\cdot\epsilon_3-2\;\not{q}\,\epsilon_q\cdot\epsilon_3\,p_1\cdot\epsilon_k+2\;\not{\epsilon}_q\,q\cdot\epsilon_3\,p_1\cdot\epsilon_k) \\
&+\Big(\tfrac{Q_2}{p_2\cdot q}-\tfrac{Q_1}{p_1\cdot q}\Big)\;\not{\epsilon}_3\,p_1\cdot\epsilon_k\;\;\not{\epsilon}_q\;\not{q}\Big] \\
&+\Big(\tfrac{Q_1Q_{1,color}}{p_1\cdot(q+k)-q\cdot k}-\tfrac{Q_2Q_{2,color}}{p_2\cdot(q+k)-q\cdot k}\Big)\;\epsilon_q\cdot\epsilon_k\;\not{\epsilon}_3 \\
&+\text{terms with factors }\not{k}\;\not{q},\;q\cdot k,\;k\cdot\epsilon_q,\text{ or }q\cdot\epsilon_k\;\;\Big\}\,(1-\gamma_5)\,u(p_1)
\end{aligned} \tag{2.8}$$

where j is any fixed number $j \in [1,3]$, and $\epsilon_3 = \epsilon^*(p_3)$, etc. $Q_{i,color}$ refers to the SU(3) Clebsch-Gordan coefficients; in this case $Q_{1,color} = Q_{2,color}$. Notice that we have arranged the expression to show u-$\bar{d}$ crossing symmetry.

First we consider the electromagnetic radiation zero. It is evident that the amplitude (2.8) vanishes in the photon null zone defined by

$$\frac{Q_i}{p_i \cdot q} = \frac{Q_j}{p_j \cdot q}, \quad \text{all } i,\ j \tag{2.9}$$

As before, this is all we need; (2.9) forces $k \propto q$ and in turn this implies

$$\frac{Q_i}{p_i \cdot k} = \frac{Q_j}{p_j \cdot k}, \quad \text{all } i,\ j \tag{2.10}$$

Second, we consider the chromodynamic radiation zero. We are reminded that there are zeros associated with any gauge group when the corresponding massless gauge bosons are emitted [5, 14]. In this reaction, we can think of two ways its tree amplitude can vanish: electromagnetic interference and chromodynamic interference. Instead of thinking of the gluon as just another particle (electrically neutral) produced along with the photon, let us consider it as the radiation due to the color charges (some of which are zero; indeed, the photon is now the "neutral" massless co-produced particle that must be parallel to the gluon). The analogous zeros for massless gluon radiation depend on the color charges. The color null zone is defined by

$$\frac{Q_{i,color}}{p_i \cdot k} = \frac{Q_{j,color}}{p_j \cdot k}, \quad \text{all } i,\ j \tag{2.11}$$

But these demand that the colorless W-boson be massless (which it is not), and that all particles be parallel, a singularly uninteresting limit. One can show, however, that the

amplitude (2.8) does have this unphysical zero.

C. Counterexamples: Compton

A question about Compton amplitudes leaps to mind when radiation zeros are studied. The null zone for photon-electron elastic scattering, for example, is easily seen to be the forward zero-momentum-transfer limit. But it is well-known that the forward amplitude does not vanish. Why is there no amplitude zero in this physical limit?

This amplitude can be rearranged as

$$
\begin{aligned}
M = i\bar{u}(p_2) \quad & \Big[\Big(\tfrac{Q_1}{p_1\cdot q_1}Q_2 - \tfrac{Q_2}{p_2\cdot q_1}Q_1\Big) \not\epsilon_2^*(\not\epsilon_1 \not q_1 - 2p_1\cdot\epsilon_1) \\
& -\tfrac{Q_1Q_2}{p_2\cdot q_1}(\not\epsilon_1\, q_1\cdot\epsilon_2^* + \not\epsilon_2^*\, q_2\cdot\epsilon_1)\Big]\, u(p_1) \\
& +i\tfrac{Q_1Q_2}{p_2\cdot q_1}\,\bar{u}(p_2)\not q_1\, u(p_2)\,\epsilon_1\cdot\epsilon_2^*
\end{aligned}
\tag{2.12}
$$

where the last term does not vanish under the conditions in (2.6). (Recall that they force $q_1 \propto q_2$ so that $q_1\cdot\epsilon_2 = 0$, etc.)

And this is not an electron spin effect; the forward amplitude is nonzero for photon-boson scattering as well. The amplitude for Compton scattering of scalar particles has a similar term (now arising from the seagull graph) which is not zero in the null zone

$$
\begin{aligned}
M = \quad & 2i\left(\tfrac{Q_2}{p_2\cdot q_1}\,Q_1 - \tfrac{Q_1}{p_1\cdot q_1}\,Q_2\right)\, p_1\cdot\epsilon_1\, p_2\cdot\epsilon_2^* \\
& -2i\tfrac{Q_1Q_2}{p_2\cdot q_1}\left[(p_1-q_2)\cdot\epsilon_1\, q_1\cdot\epsilon_2^* - q_2\cdot\epsilon_1\, p_2\cdot\epsilon_2^*\right] \\
& +2iQ_1Q_2\,\epsilon_1\cdot\epsilon_2^*
\end{aligned}
\tag{2.13}
$$

The reason the Compton amplitudes are not null in the null zone lies in the forward limit where there is no momentum transfer. We turn our attention to the general proof in order to understand, among other things, this exceptional case.

III. The General Result

We can understand where there are multi-photon zeros in gauge theoretic tree amplitudes by appealing to a general radiation interference theorem for single-photon zeros and certain neutral particle lemmas associated with it [5]. In this section, we revisit the proof of those lemmas to show two things: First, how the examples of the previous section fit into the arguments, with the neutral particles identified as additional photons. Second, how the proof is readily generalizable to multi-boson zeros for the emission of other massless gauge bosons. This opportunity lets us reference also a larger, unpublished version [15] of the previous work.

The presence of a radiation zero for single-photon emission means that it possible to rewrite the tree amplitude in a factorized form, really, a sum of factored terms in one-to-one correspondence to the set of independent conditions, such as those in (2.2). A radiation representation has been found in [5] where the formulas are organized according to the original vertices in the RsourceS graphs to which the photons would be attached. The radiation amplitude can be written as a sum of vertex attachments with coefficients related to the rest of the original source graph. Each vertex term V has a radiation representation of the form

$$M_\gamma(V) \;=\; \sum_{i=1}^{n} \delta_i p_i \cdot q \Delta_{ij}(Q)\Delta_{ik}(\delta J) \tag{3.1}$$

with any fixed choice for j and k, and the definition

$$\Delta_{ij}(x) = \frac{x_i}{p_i \cdot q} - \frac{x_j}{p_j \cdot q} \tag{3.2}$$

It is supposed that there are n internal and external legs on the vertex, and J_i is the product of the photon-emission current for the i^{th} leg and the remaining factors of the original vertex amplitude. For other gauge groups, the charges Q_i refer to the Clebsch-Gordan coefficients coupling an incoming particle to an outgoing particle through the gauge boson vertex. See Secs. VI and X of [5] for more detail.

We can use the above development for a multi-photon argument, by considering one of the source's external legs to correspond to another photon. (Although neutral internal lines are not of interest to us, they do not spoil radiation zeros anyway.) It might appear that there is no zero if one of the original external particles r has zero charge, $Q_r = 0$. One term in a Δ factor in (3.1) is eliminated and hence that factor will not vanish in the null zone. But this is not the whole story.

Looking at the terms in the Δ factors in (3.1), we still get zero if, in addition to $\Delta_{ij}(Q) = 0$ (for $j, k \neq r$) we have $p_r \cdot q = 0$ and $J_r = 0$ in the null zone. So the neutral particle must be massless (which is fine for photons!) and travel parallel to the photon (which are just the conditions derived [5, 15] from the zero-charge limit, $Q_r \to 0$, of the null zone equations, forcing $p_r \cdot q \to 0$). Furthermore J_r can vanish when $p_r \propto q$ for a massless vector neutral particle r if it is coupled to a conserved current (which is also fine for photons!) in a "nonforward" direction (explained below). To understand this vanishing for a given photon attachment, let us go over the various current contributions for $p_r \propto q$. The convection current $p_r \cdot \epsilon$ is clearly zero, as is the contact current which involves the contraction $p_r^\alpha \, \omega_{\alpha\beta}$ where $\omega_{\alpha\beta} = q_\alpha \, \epsilon_\beta - \epsilon_\alpha \, q_\beta$. The vector spin current $\omega_{\alpha\beta}\eta_r^\beta = q_\alpha \, \epsilon \cdot \eta_r$ contains the factor q_α

(which is to be contracted into the conserved current vertex source of the vector particle). This factor is proportional to the momentum transferred to the vertex, $\Delta p = p_r \pm q$ for photon emission from a particle in the final/initial state. Thus, if the momentum transfer Δp is nonzero ("nonforward scattering"), the vector spin current contribution vanishes by current conservation. When the momentum transfer is zero, however, we no longer have any proportionality relation, the vector spin current contribution is not zero, and neither is the amplitude.

The point is that if the null zone corresponds to forward scattering of massless vector particles (like photons) then the amplitude is not null. The last terms in the Dirac and scalar Compton amplitudes, (2.12) and (2.13), respectively, do not vanish under the null zone conditions, and exemplify the vector currents of which we

The lemmas in the references [5, 15] tell us that an arbitrary number of neutral external particles can coexist with a radiation zero, as long as they are all massless and all travel parallel to the photon, and hence to each other. We can take the special case of their all being photons, each certainly coupled to a conserved current, and we merely need to avoid forward scattering limits where the picture is of a subset of initial photons turning into a subset of final photons without a change in the overall momentum of the photon "pack". Considering only final photons, for example, eliminates this problem.

The resulting null zone is consistent with the zero-charge limit of the general null zone conditions. And in fact it is often useful to think of the final (or initial) multi-photon subset as a massless composite particle.

We cannot write radiation amplitudes for multi-boson emission in which the zeros are made manifest by a series in Δ_{ij} factors. As seen in the examples (2.5) and (2.8), some terms do not have these factors, yet vanish in the null zone by virtue of their momentum and current dependence. These terms are again related to the currents J_r analyzed above.

We can, however, establish simple forms in the limit where all photons have the same momentum. This is related to the all-orders solution for an external plane-wave field coming up next.

IV. The More General Result: An External Field

It has been pointed out previously [12] that multiphoton zeros follow from a decoupling theorem for the scattering of a system of particles immersed in an exernal electromagnetic plane wave. In this section we review and elaborate upon the details showing this connection, and we use one of the examples in Sec. II to demonstrate the result.

For a particle with charge Q and mass m coupled to an external electromagnetic plane wave $A_\mu = A_\mu(n \cdot x)$, $n^2 = 0$ (gauge $n \cdot A = 0$), the wave functions for spins 0, 1/2, 1 can all be written in the form [12, 13]

$$\Psi(x) = ULT\chi(x) \tag{4.1}$$

where χ is the free solution and U,L,T are local gauge, Lorentz, and displacement transformations, respectively. Explicitly, we have for spins $\{0,\ 1/2\}$ and a free plane wave,

$$\chi_p(x) = e^{-ip\cdot x}\,\{1;\ \omega(p)\} \tag{4.2}$$

where $p^2 = m^2$, $\not{p}\omega = m\omega$, and

$$
\begin{aligned}
&L = \{1; 1 + \tfrac{Q}{2n\cdot p} \not{n} \not{A}\} \\
&U(\theta) = e^{i\theta}, \qquad\qquad \theta = \tfrac{Q^2}{2n\cdot p} \int^{n\cdot x} dz\, A^2(z) \\
&T(d) = e^{-ip\cdot d}, \qquad\qquad d^\mu = \tfrac{Q}{n\cdot p} \int^{n\cdot x} dz\, A^\mu(z)
\end{aligned}
\tag{4.3}
$$

The spin-one results can be found in the references [12, 13].

Consider initially the scattering of a system of particles with no external field. The tree amplitude is

$$
\mathcal{T} = \prod_V \prod_I \int dp_I\, D(p_I) V(k) \tag{4.4}
$$

with internal propagators $D(p_I)$ and vertex factors $V(k)$ [k legs and including delta functions $\delta(\sum^k p_i)$]. If we turn on the external electromagnetic field A, the internal and external legs of the tree amplitude (4.4) are altered according to the Fourier transform of (4.1) changing the δ-functions to

$$
\delta_{ext} = \prod_{j=1}^{k} (ULT)_j \delta(\sum_{i=1}^{k} p_i) \tag{4.5}
$$

where it is understood that we replace $n \cdot x$ by $in \cdot \partial/\partial p_j$ in the $(ULT)_j$. Supplementary changes for vertices with derivative couplings are discussed in [12, 13], but in any case (4.5) helps us understand the changes in particle momenta due to the external field. For a monochromatic external wave, $A_\mu = 2Re\,(N\epsilon_\mu e^{-iq\cdot x})$, with frequency ω and momentum $q = \omega n$, we see how harmonics arise through the identity

$$
(e^{\pm q\cdot\partial/\partial p})^l \delta(p) = \delta(p \pm lq) \tag{4.6}
$$

To see the generalized radiation zero, we ask that the conditions analogous to (2.6) be satisfied

$$\frac{Q_i}{n \cdot p_i} = \text{same for all external particles } i \tag{4.7}$$

The effect of $\partial/\partial p_j$ on the delta function in (4.5) is independent of j, implying the various θ_j and d_j of (4.3) are also independent of j. From charge conservation, Lorentz invariance, and momentum conservation, all the phases (group parameters) cancel out:

$$\prod_{j=1}^{k} (ULT)_j = 1, \text{ in the "null zone" defined by (4.7)} \tag{4.8}$$

We see that the external field effects have disappeared; the field is decoupled in the null zone. Even though it may have been kinematically allowed for the particle system to evolve to some final state under the influence of the external field, the probability amplitude for that is zero!

An expansion order-by-order of (4.5) in the various charges of the particles is in one-to-one correspondence with the sequence of amplitudes for n collinear photons. For example, attaching n photons with the same momentum q (and polarization) to a given leg in all possible ways, remembering the seagull graphs for scalars, leads to an exponential form when summed over n. In this way, we see the connection between the zeros for an n-photon amplitude and the decoupling theorem. When the exponentials collapse to unity, every such amplitude is zero. To generalize to photons with different polarizations, we can replace QA by $Q_1A_1 + Q_2A_2 + ...$ in (4.3) and, as long as the different external fields are collinear with respect to their null vectors n_i, the forms (4.1) and (4.5) continue to hold. The expansion in

charge produces now the more general n-photon amplitudes, with independent polarization for each photon. Again a decoupling theorem exists and implies the higher-order radiation zeros amplitude-by-amplitude.

It is satisfying to compare an expansion of (4.5) with our examples. Equation (2.5) in the limits $q_1 = q_2 = q$ and $\epsilon_1 = \epsilon_2 = \epsilon$ is

$$\begin{aligned} M \quad &= g \sum_i \left(\tfrac{Q_i}{p_i \cdot q} - \tfrac{Q_j}{p_j \cdot q} \right) \delta_i p_i \cdot \epsilon^* \sum_k \left(\tfrac{Q_k}{p_k \cdot q} - \tfrac{Q_l}{p_l \cdot q} \right) \delta_k p_k \cdot \epsilon^* \\ &- \tfrac{1}{2} g (\epsilon^*)^2 \sum_i \left(\tfrac{Q_i}{p_i \cdot q} - \tfrac{Q_j}{p_j \cdot q} \right) \delta_i Q_i \end{aligned} \tag{4.9}$$

This checks perfectly against the second-order term; the first line comes from two powers of "d terms", and the second line corresponds to one power of "θ terms", refering to the nomenclature in (4.3).

V. Experiments and Discussion

In this last section we discuss reactions involving the production of multiple photons, gluons, or supersymmetric partners thereof, which are well approximated by tree amplitudes, and where their tree amplitudes have radiation amplitude zeros (RAZ). We often have in mind the possibility that the zeros may be sensitive to fundamental particle parameters.

There have already been some limits set on the W magnetic moment parameter κ from $W\gamma$ and radiative W decay at CDF (Fermilab) and UA2 (CERN). The results [16] are

$$\begin{aligned} &-2.4 \leq \kappa \leq 3.7 \qquad\qquad \text{(CDF)} \\ &-3.1 \leq \kappa \leq 4.2 \qquad\qquad \text{(UA2)} \end{aligned} \tag{5.1}$$

So far, since the number of events are limited [17], only the total number of events have been used to obtain these limits. The new run at CDF and also D0 will obtain many more events and then one should be able to obtain an angular distribution and, hopefully, see the RAZ. For a related discussion, see [18]. Also, the rapidity correlation study by Baur et al. [7] is a new and effective tool in the radiation zero analysis.

Of course the RAZ occurs only if $\kappa_W = 1$, or $g_W = \kappa_W + 1 = 2$. Thus this is a test of the Standard Model (SM) in which $\kappa_W = 1$ (plus radiative corrections). Recently Brodsky and Hiller [19] have shown that a composite particle has in general non-standard magnetic and quadrupole moments. However, in the limit of zero radius the moments take their SM values. This has been shown for spin 1. The spin 1/2 case was treated earlier [20]. Thus the RAZ in the reactions described previously and in what follows constitute a test of the compositeness of the W boson.

Consider first the 2γ double bremsstrahlung process

$$Q_1 + Q_2 \rightarrow Q_1 + Q_2 + \gamma + \gamma \tag{5.2}$$

This process is being studied by Ward et al. [21]. As a check of this calculation, one may impose the null-zone conditions described in Sec. II, irrespective of whether the zero is physical or not. The differential cross section should then vanish. If it does not, there is an error in the calculation. Such computational checks are a useful feature of the presence of radiation zeros, particularly in the higher-order QED and QCD calculations that have become increasingly relevant to experimental analysis.

We come back now to the processes

$$W \to d\bar{u}\gamma g \tag{5.3}$$

$$d\bar{u} \to W\gamma g \tag{5.4}$$

We have described earlier in Sec. II that these reactions have zeros essentially at the same places as the original reactions without the gluons, but now with the gluon and photon traveling together. These gluon processes could be seen in [22]

$$pp(\bar{p}) \to W\gamma gX \to (e,\mu)\nu\gamma gX \tag{5.5}$$

$$pp(\bar{p}) \to WX \to (e,\mu)\nu\gamma gX$$

where a sharp dip should persist. These again occur only if $\kappa_W = 1$, or $g_W = 2$ and, therefore, are a test of the SM. Here we must be able to distinguish gluon jets from quark jets and remove the $q\bar{q}$ background. One could imagine tagging gluon jets with photons, and thereby verifying the consistency of jet identification algorithms. One would look for photons inside gluon jets and find no events when the zero conditions are satisfied. Such an experiment, although difficult would be very interesting.

The difficulty of detecting the photon and jet together has been emphasized by Diakonos et al. [23]. In their recent paper, they refer to the neutral particle extension [5] of the radiation zeros, noting that the zero arising when the photon and gluon are parallel (and at the original RAZ magic angle) is a powerful check on the matrix element calculation. Although they are much less sanguine about the experimental consequences, there is the possibility that the

recent approach by Baur et al. [7] may be adapted to the W-photon-gluon final state, and improve the signal to noise.

One could also look at HERA (DESY) for the process [9, 10, 11]

$$e^+u \to e^+u\gamma g \tag{5.6}$$

This could be seen in

$$e^+p \to e^+p\gamma gX \tag{5.7}$$

where a dip should persist. In the corresponding process

$$e^-p \to e^-p\gamma g \tag{5.8}$$

the zero is washed out [10, 11].

In addition, we can consider supersymmetric versions of the radiation zeros [24, 25]. For example, in

$$\chi^+ \to \chi^0 u\bar{d} \tag{5.9}$$

a RAZ occurs in the supersymmetric limit, when

$$\tan\beta = 1 \tag{5.10}$$

and the masses are equal. In the context of this paper, it is to be noted that adding photons or gluons, or their supersymmetric partners, again does not subtract the zeros.

Recently, Ohnemus and Stirling [26] considered the process

$$pp \to W^\pm\gamma\gamma X \tag{5.11}$$

which is obtained from the elementary processes

$$q + g \rightarrow W + \gamma + q(\rightarrow \gamma X) \tag{5.12}$$

$$q + \bar{q} \rightarrow W + \gamma + g(\rightarrow \gamma X) \tag{5.13}$$

$$q + \bar{q} \rightarrow W + g(\rightarrow \gamma X) + g(\rightarrow \gamma X) \tag{5.14}$$

$$q + \bar{q} \rightarrow W + q(\rightarrow \gamma X) + \bar{q}(\rightarrow \gamma X) \tag{5.15}$$

$$q + g \rightarrow W + q(\rightarrow \gamma X) + g(\rightarrow \gamma X) \tag{5.16}$$

$$g + g \rightarrow W + q(\rightarrow \gamma X) + \bar{q}(\rightarrow \gamma X) \tag{5.17}$$

These processes were considered as background to the search for the Higgs boson via associated production with W bosons. This process

$$pp \rightarrow W + H(\rightarrow \gamma\gamma) + X \tag{5.18}$$

provides a very clean signature and could be used at the SSC or the LHC to find the Higgs. Process (5.13) has the physical RAZ we have been talking about. One could hope that a dip persists in (5.11); we note again the recent work of Baur et al. [7]. A rough estimate suggests that one could obtain 200 such events at the SSC.

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References

[1] K.O. Mikaelian, M.A. Samuel, and D. Sahdev, Phys. Rev. Lett. **43**, 746 (1979).

[2] R.W. Brown, D. Sahdev, and K.O. Mikalian, Phys. Rev. D **20**, 1164 (1979).

[3] S.J. Brodsky and R.W. Brown, Phys. Rev. Lett. **49**, 966 (1982).

[4] M.A. Samuel, Phys. Rev. D **27**, 2724, (1983).

[5] R.W. Brown, K.L. Kowalski and S.J. Brodsky, Phys. Rev. D **28**, 624 (1983).

[6] M.A. Samuel, N. Sinha, R. Sinha and M.K. Sundaresan, Phys. Rev. D **44**, 2064 (1991).

[7] Very recently, it has been shown that rapidity correlations involving the photon and the charged decay lepton and not requiring the reconstruction of the parton center of mass frame, display a pronounced dip corresponding to the radiation zero: U. Baur, S. Errede, and G. Landsberg, FSU-HEP-930727, to appear in the Proceedings of the Workshop "Physics at Current Accelerators and the Supercollider", Argonne National Laboratory, June 2-5, 1993. See also U. Baur and D. Zeppenfeld, Nucl. Phys. **308B**, 127 (1988); U. Baur and E.L. Berger, Phys. Rev. D **41**, 1476 (1990).

[8] T.R. Grose and K.O. Mikaelian, Phys. Rev. D **23**, 123 (1981).

[9] C.L. Bilchak, J. Phys. G: Nucl. Phys. **11**, 1117 (1985).

[10] G. Couture, Phys. Rev. D **39**, 2527 (1987).

[11] J. Reid, G. Li, and M.A. Samuel, Phys. Rev. D **41**, 1675 (1990).

[12] R.W. Brown and K.L. Kowalski, Phys. Rev. Lett. **51**, 2355 (1983).

[13] R.W. Brown and K.L. Kowalski, Phys. Rev. D **30**, 2602 (1984).

[14] C.J. Goebel, F. Halzen, and J.P. Leveille, Phys. Rev. D **23**, 2682 (1981).

[15] R.W. Brown, K.L. Kowalski, and S.J.Brodsky, Report No. Fermilab-82/102, 1982 (unpublished).

[16] M.A. Samuel, G. Li, N. Sinha, R. Sinha and M.K. Sundaresan, Phys. Lett. **280B**, 124 (1992).

[17] J. Aitti et al., Phys. Lett. **277B**, 194 (1992);
M. Timko, "Status of WGamma Search at CDF", APS Meeting (Washington, DC, April 1990) unpublished.

[18] J. Reid and M.A. Samuel, Phys. Rev. D **39**, 2046 (1989).

[19] S.J. Brodsky and J.R. Hiller, Phys. Rev. D **46**, 2141 (1992).

[20] S.J. Brodsky and S.D. Drell, Phys. Rev. D **22**, 2236 (1980).

[21] B. Ward, private communication.

[22] A careful assessment of QCD corrections to the original radiation zero has been given by J. Smith, W.L. van Neerven and J.A.M. Vermaseren, Z. Phys. C **30**, 621 (1986); J. Smith, D. Thomas and W.L. van Neerven, Z. Phys. C **44**, 267 (1989); S. Mendoza, J. Smith and W.L. van Neerven, Phys. Rev. D **47**, 3913 (1993).

[23] F.K. Diakonos et al., Phys. Lett. **303B**, 177 (1993).

[24] R.W. Brown and K.L. Kowalski, Phys. Lett. **144B**, 235 (1984).

[25] D. DeLaney, E. Gates and O. Tornqvist, Phys. Lett. **186B**, 91 (1987).

[26] J. Ohnemus and W.J. Stirling, Phys. Rev. D **47**, 336 (1993). See also E.N. Argyres et al., Phys. Lett. **280B**, 324 (1992).

Figure Captions.

1. Lowest-order diagrams for decay of a scalar particle through a single scalar interaction into $n-1$ scalar particles and a photon.

2. Diagrams for a photon attached to a sample tree source graph with one internal line.

3. Diagrams for the radiative decay process where quark-antiquark annihilation leads to a W-boson plus a photon and a gluon.